Selecting SAT Encodings for Pseudo-Boolean and Linear Constraints: Preliminary Results



Felix Ulrich-Oltean, Peter Nightingale, James Walker

University of York

Felix is supported by grant EP/R513386/1 from the UK Engineering and Physical Sciences Research Council.

This Talk

An example SAT encoding

Learning encoding choices

Features

Results and conclusions

Encoding

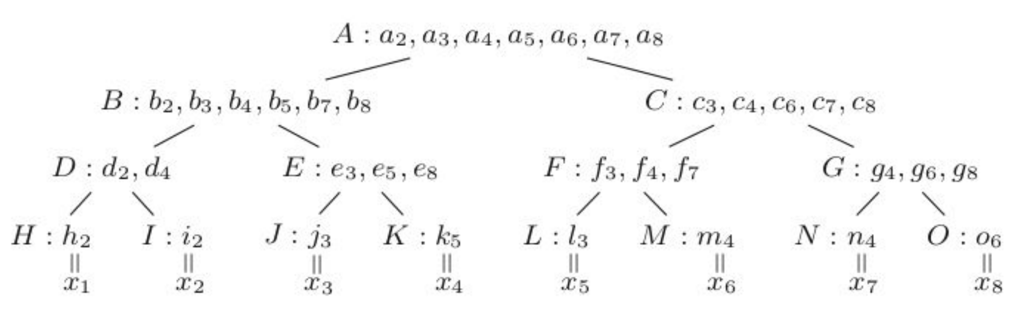
Learning

Features

Results

The talk is based on our ModRef2021 paper ***Selecting SAT Encodings for Pseudo-Boolean and Linear Constraints: Preliminary Results***. We thank the reviewers for their helpful comments.

Encoding Example



binary tree of $GT(2x_1 + 2x_2 + 3x_3 + 5x_4 + 3x_5 + 4x_6 + 4x_7 + 6x_8 \leq 7)$

$$\overline{l_{w_1}} \vee \overline{r_{w_2}} \vee o_{w_3} \quad l_{w_1} \in L.vars,\ r_{w_2} \in R.vars,\ w_3 = \min(w_1 + w_2, K+1) \tag{9}$$

$$\overline{t_w} \vee o_w \quad t_w \in L.vars \cup R.vars \tag{10}$$

$$\overline{a_{k+1}} \tag{11}$$

An extract from a SAT encoding description for a pseudo-Boolean sum constraint.

Diagrams and clauses for the “Generalized Totalizer” from *Bofill, Coll, Suy, Villaret: SAT Encodings of Pseudo-Boolean Constraints with At-Most-One Relations*, in CPAIOR 2019 https://doi.org/10.1007/978-3-030-19212-9_8

Encoding
Learning
Features
Results

Encoding a Constraint

	enc.	Q1	med	Q3	avg	t.o.	v.	cl.	g.t.
Set1	**MDD**	3.89	14.78	73	131	87	25	266	3.71
	GSWC	4.50	5.92	277	158	112	105	1076	10.01
	GGT	—	—	—	—	—	—	—	—
	GGPW	0.04	0.04	5.54	93	67	1.0	4.4	0.05
Set2	**MDD**	0.21	0.41	1.42	74	53	2.1	21	0.28
	GSWC	0.58	0.62	1.09	71	52	6.4	66	0.62
	GGT	2.42	8.83	53	132	95	1.9	120	1.53
	GGPW	0.02	0.03	3.36	89	65	0.6	2.5	0.03

Encoding

Learning

Features

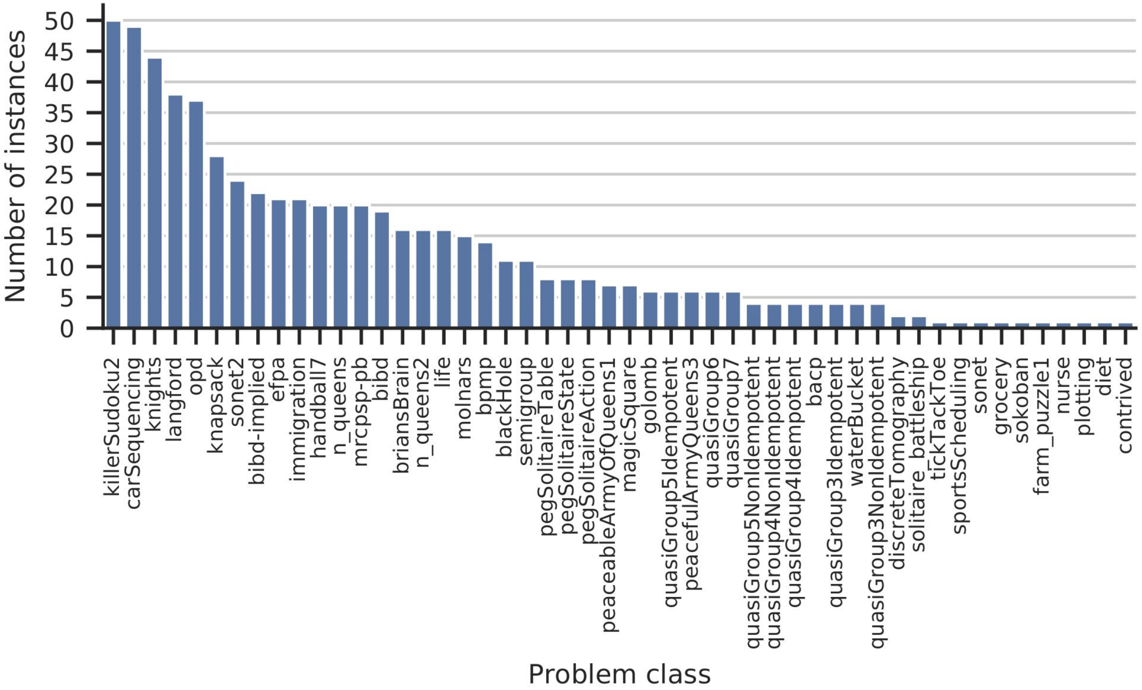
Results

Performance summary from *Bofill, Coll, Suy, Villaret: SAT Encodings of Pseudo-Boolean Constraints with At-Most-One Relations*, in CPAIOR 2019
https://doi.org/10.1007/978-3-030-19212-9_8

Experimental setup

- Savile Row has MDD, GSWC, GGPW, GGT + Tree encodings
- 5 choices for sums x 5 choices for PBs = 25 configurations
- each instance run with each configuration 5 times and the median time taken (to average out SAT solver randomness)
- timeouts set to 1 hour each for Savile Row and the SAT solver (Kissat)

Problem Corpus



Encoding

Learning

Features

Results

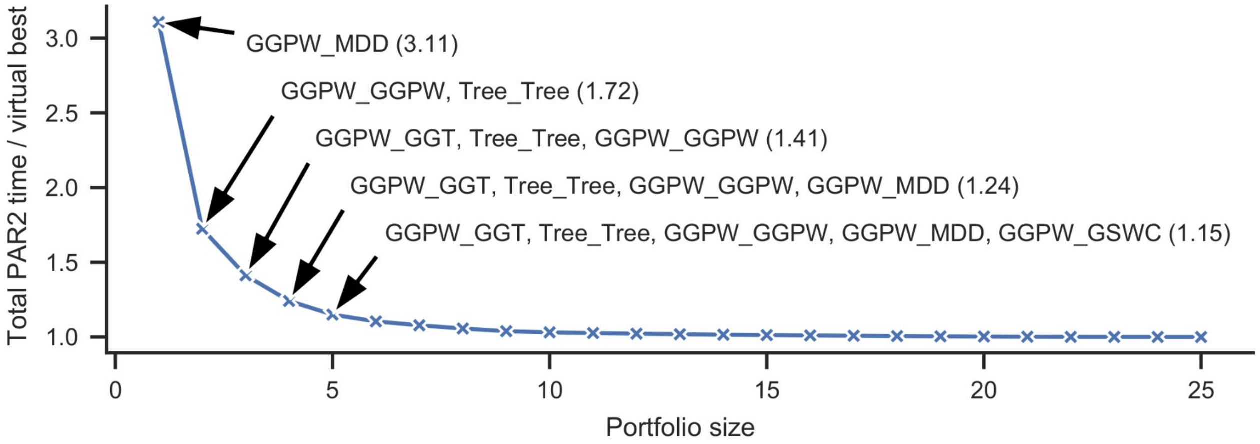
Essence Prime Models mainly from *Davidson, Akgün, Espasa, Nightingale: Effective Encodings of Constraint Programming Models to SMT.*
In CP 2020 https://doi.org/10.1007/978-3-030-58475-7_9

Pairwise Training

- random forests trained to make binary choice for each pair of configurations
- pairwise predictions give a ranking
- top configuration becomes our prediction

Pairwise voting random forests inspired by *Lindauer, Hoos, Hutter, Schaub: AutoFolio: An Automatically Configured Algorithm Selector*. In JAIR 2015
https://doi.org/10.1613/jair.4726

A complementary portfolio



Encoding

Learning

Features

Results

The virtual best PAR2 run-time on our corpus for all portfolio sizes as a multiple of the overall virtual best; the resulting portfolios (of *li_pb* configurations) are shown for sizes 1 to 5

Instance Features

- **f2f**: from `fzn2feat` tool [1]: 95 generic CSP instance features relating to constraints, variables, and their domains. Extracted by outputting FlatZinc from Savile Row, then running `fzn2feat`
- **f2fsr**: an attempt to extract the same features from Savile Row's internal model just before encoding to SAT
- **sumpb**: new pb-related features
- **combi**: *f2fsr* and *sumpb* combined

[1] *Amadini, Gabbrielli, Mauro: An enhanced features extractor for a portfolio of constraint solvers*. In SAC '14 https://doi.org/10.1145/2554850.2555114

PB-specific Features

Considered:

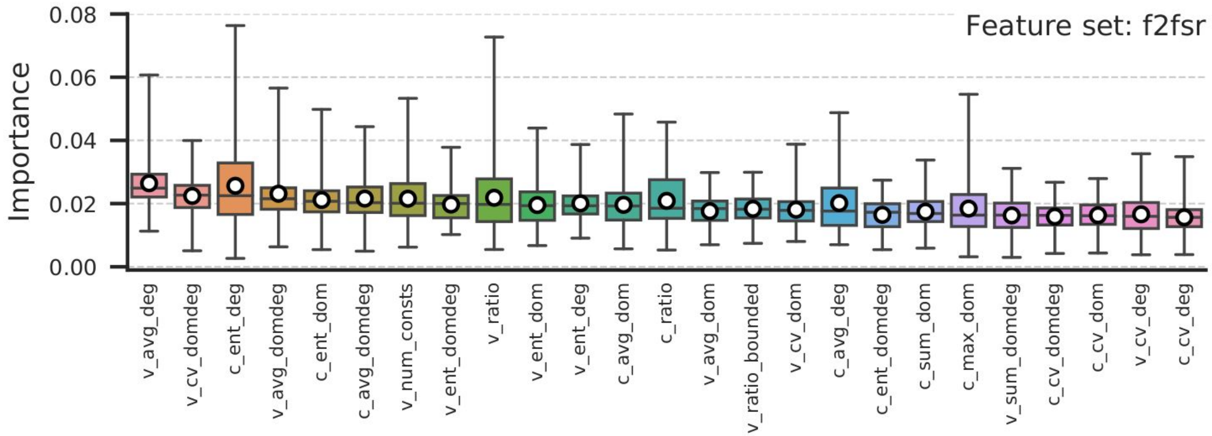
- number of PBCs (or LI)
- number of terms in constraints
- coefficients in the constraints
- number of distinct coefficients

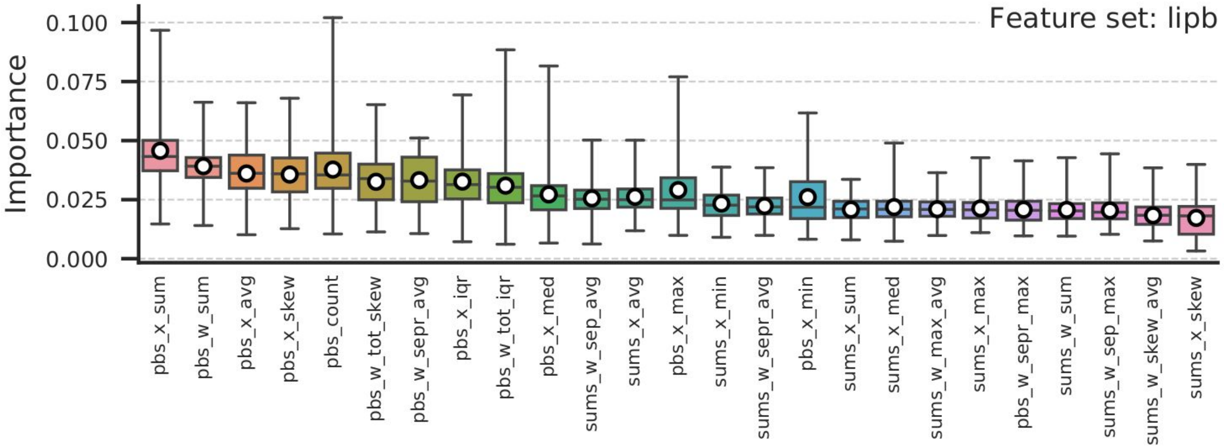
Calculated (a selection of):

- averages (arithmetic mean, median)
- spread (IQR)
- min, max, sum
- skewness (non-parametric, quartile)
- Shannon's entropy

Full details in our ModRef2021 paper, *Selecting SAT Encodings for Pseudo-Boolean and Linear Constraints: Preliminary Results*

Feature Importance





Encoding

Learning

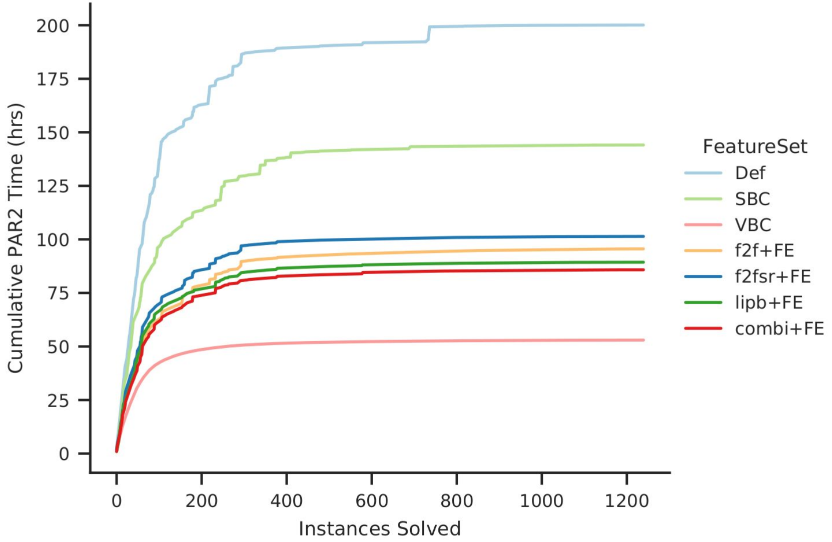
Features

Results

Evaluating performance

Benchmarks				Predicted				Predicted + FE Time			
VBC	VWC	SBC	Def	f2f	f2fsr	lipb	combi	f2f	f2fsr	lipb	combi
1.00	6.66	2.72	3.78	1.76	1.89	1.67	1.60	1.80	1.91	1.69	**1.62**

Total PAR2 times over the 10 test sets **as a multiple of the virtual best configuration** time. We show the times for the virtual best (VBC), virtual worst (VWC), single best (SBC), and default (Def) configurations, followed by timings for our predictions.



PAR2 time over 10 test sets, sorted (by VBC solving time descending)

Findings and Future Work

For our corpus:

- ML can outperform the single best encoding
- good encoding for PBs more important than for sums

In the future:

- extend to a broader benchmark of problems
- apply to other encoding choices
- consider at-most-one groups